The Evolutionary Brexit Game: Uncertainty and Location Decision

Izak Carlos Silva* Douglas Sad Silveira † Silvinha Vasconcelos ‡

ABSTRACT

This article proposes a new methodology to analyze the strategic decision about firms' location choice faced with the uncertainty surrounding Brexit. For this, we combine the Evolutionary Game Theory (EGT) approach with input-output analysis. We present a study of case to empirically describe and evaluate our model. In this sense, we consider that firms are competing in two different sectors: (a) crop and animal production, hunting and related service activities; (b) financial service activities, except insurance and pension funding. By doing so, the European Union was separated into two strategic regions: United Kingdom and rest of European Union. To decide where to locate, firms consider the following exogenous factors: (i) potential market; (ii) local productive interdependence; (iii) labor costs and (iv) displacement cost. To generate the results, we create hypothetical Scenarios, in which firms can assign specific weights to each of these factors in the decision-making process. The results suggest that the occurrence of Brexit can be determinant in the location decision of firms according to the sector of action and the weight given to the factors associated with the production process. In traditional sectors and considering an environment of uncertainty, firms tend to seek unsaturated markets. On the other hand, in sectors associated with services, the greater the uncertainty, the greater the likelihood that firms will move.

Key-words: Location Decision; Brexit; Input-Output Analysis.

JEL Classification: R30, C70, F15.

*PhD student at the Graduate Program in Economics at UFJF.

†PhD student at the Graduate Program in Economics at UFJF.

‡Full Professor of the Graduate Program in Economics of FEA/UFJF.

1 Introduction

Firm's strategic location decision is discussed in many fields of Economic Literature. From the Regional Economics perspective, the location decision is based on the balance between a complex relation of attraction and repulsion of consumers and firms (LÖSCH, 1940; HOOVER, 1948). In this way, the economic activity to be developed in a given region can be determined by minimizing costs [1] or by maximizing market potential [2].Recently, through the New Economic Geography approach, the location decision choice made by the firms has been handled jointly by these two perspectives, as can be seen in Krugman e Venables (1990), Krugman (1991).

By it turns, the Industrial Organization (IO) framework is mostly concerned with the theoretical understanding of the competition nature in markets when firms strategically decide where to locate. According to Fujita e Thisse (2002), Silva, Mota e Grilo (2015), by introducing interregional labor mobility as one of the central aspects in the location decision of firms, it is possible to establish a dialogue between the regional economy, industrial organization, international trade and theories of growth and economic development.

According to Rocha et al. (), despite the vast literature[3] that has been established about location decision in these fields of study, there are few studies about spatial competition[4] lying on the interface of evolutionary game theory (EGT) and regional science, since the EGT models have been applied mainly in IO researches. Although it may still be in its infancy, this theme should attract more interest from regional analysis, because the competitive locational problem emerges as a prototype of many economic situations involving dynamically interacting decisions in which firms can learn with their own choices over time.

A fact that illustrates well the circumstances presented above is the issue involving Brexit. As presented in Dhingra et al. (2016), the possible withdrawal of the United Kingdom (UK) from the European Union (EU) may generate an environment of uncertainty and economic instability. To be more precise, Brexit may imply uncertainties in the locational decision of the firms. Gandolfo (1998) argue that the end of common markets could harm the customs union and reduce competition and the scale of production. Another

[1] See (DUNN, 1954; SCHWEIZER; VARAIYA, 1976; BECKMANN, 1972; ISARD, 1956; ALONSO, 1964).

[2] See (OGAWA; FUJITA, 1980; HENDERSON, 1974; HENDERSON, 1991; HOTELLING, 1990).

[3] To illustrate the interface of these fields, we can mention d'Aspremont, Gabszewicz e Thisse (1979), who considered a slightly modified version of Hotelling's model, in which exists a tendency for both sellers to maximize their differentiation. This constitutes a counterexample to the conclusions originally presented by Working e Hotelling (1929). By its turn, Gabszewicz e Thisse (1992) provided the framework for a spatial competition model and the location of firms

[4] In Chan (2001), Fischer e Nijkamp (2014)and Wilson (2014) there is a useful compendium of spatial analysis techniques which points out the commonalities among models used to locate facilities one at a time and to forecast the economic development pattern in an entire region.

possible consequence is an increase in legal and economic insecurity.

In this way, the exit process from the United Kingdom of the European Union may imply reconsideration of the optimal location by firms according to productivity differentials, factor prices (JONES; KIERZKOWSKI; LURONG, 2005), regional characteristics, size of internal markets and regulatory issues (LUNA; DIETZENBACHER; HEWINGS, 2009).

Thus, under such conditions, EGT may bring some insight into the behavioral pattern of firms' location decision. While the traditional theory of games requires that players have a very high level of rationality, the EGT model has been used to successfully explain a number of aspects of agents' behavior. More specifically, EGT may accomplish better success in describing and predicting the choices of locational decisions, since it is better equipped to handle the weaker rationality assumptions[5].

Considering the existent literature and the lack of contributions of EGT to spatial theories as mentioned before, in this work we develop a new model that considers the projection made from the Regional Economic analysis for dynamically guiding firms to the Evolutionary Stable Strategy (ESS), i.e., to the optimal strategy location decision in the long term, considering the possible occurrence of Brexit.

The idea behind an ESS is to ensure that a so called mutant strategy will not be able to dominate a competitive environment that embraces the incumbent strategy. Furthermore, as explained in Friedman (1991), the EGT provides a refinement of the dynamic approach applied in the traditional game theory, allowing an inference about which Nash Equilibrium (NE) corresponds to an ESS.

Therefore, we present a study of case with the objective of offering a reasonable explanation about firms' location patterns in the European Union countries due to the possible occurrence of Brexit. In order to aim this goal, we evaluate the behavior of firms under a new methodology. We consider that firms are competing in two different sectors: (a) crop and animal production, hunting and related service activities (Sector 1); (b) financial service activities, except insurance and pension funding (Sector 41). In the evolutionary game presented here, we built the payoffs based on information received from the input-output methods using the WIOD database.

With the purpose of capturing the effects of exogenous variables that can affect strategic location decisions, the payoffs incorporate weighted factors, whose weights vary with each Scenario we built. Since there are multiple possible outcomes that vary with each weight we consider, our results show that the Scenario and its inherent uncertainty may affect the strategic decisions when the outputs cannot be predicted.

On this matter, starting from an evolutionary perspective, the main objective of this paper is to offer a reasonable contribution to the comprehension of the strategic behavior

[5] A small sampling of topics that have been analyzed from the evolutionary perspective includes altruism (GINTIS et al., 2003) and behavior in public goods game (CLEMENS; RIECHMANN, 2006).

pattern of firms' spatial competition. Thus, we assume in our analysis representative multinational firms that have to decide between United Kingdom (UK) and rest of European Union (EU) whether to locate. To make that decision, besides assigning probability to the occurrence of Brexit, the following factors are considered: (i) potential market; (ii) local productive interdependence; (iii) labor costs and (iv) Displacement Costs. In order to reach our purpose and develop the discussion proposed in this introduction, this article is subdivided into 4 more sections. Section 2 brings our methodology. The Evolutionary Brexit Game is presented in section 3. The results will be analyzed and discussed in section 4. Finally, section 5 will bring a brief conclusion and some important remarks to be addressed in future research.

2 Methodology

2.1 Strategic Elements Evaluated in the Location Decision

In order to develop the methodology, we will consider a game involving two types of firm. The $TypeA$ firm is situated in the rest of the European Union countries (EU) and assesses the possibility of migrating to the United Kingdom (UK) in the face of the uncertainty surrounding Brexit. The $TypeB$ firm is situated in (UK) and assesses the possibility of migrating to (EU).

The payoffs of the game that mimic the decision-making of the firms upon Brexit occurrence were constructed based on the results of the regional input-output matrix, made available by the WIOD for 2014, with sector opening of 43 countries and 56 sectors, as well as an adjustment account called "Rest of the World".

In addition, the 43 countries were aggregated so that the interregional input-output matrix used in this paper contains the same 56 sectors for three regions: United Kingdom (GBR), Rest of Europe Union (RoEU) and Rest of the World (RoW).

2.2 Market Potential

Firms see the maximization of the market area as an important factor in the location decision (CHRISTALLER, 1933; OGAWA; FUJITA, 1980; HOTELLING, 1990). It can be measured by the input-output model, expressed as in Miller e Blair (2009) by:

$$X = (I - A)^{-1}F \tag{1}$$

where $X_{n\times 1}$ is a column vector with n rows, $I_{n\times n}$ is a dimension identity matrix n by n, $SA_{n\times n}$ is a matrix of technical coefficients, whose size is $n \times n$, $F_{n\times 1}$ is the column vector of final demand and $(I - A)^{-1}$ is known in the economic literature as the inverse matrix of Leontief, here denoted by B.Thus, we can rewrite equation (1) as follows:

$$X = BF \tag{2}$$

From 2 we can represent the model in its interregional form, as (JOHNSON; NOGUERA, 2012), by:

$$B^D X + F^D = X \tag{3}$$

$$B^M X + F^M = X \tag{4}$$

$$\mu B^D + \mu B^M + B_v = \mu \tag{5}$$

where B represents a $n \times n$ matrix of the coefficients for domestic production. F is a final demand vector, of size $1 \times n$, which includes the gross fixed capital formation, public and private consumption, and exports. M is a import vector $n \times 1$. B_v is a vector of size $1 \times n$ which indicates the rate of value added on the total production for the sector i from the country j. μ is a unit vector of size $1 \times n$. The overwritten D indicates that the variables are domestic and the overwritten M indicates that the variables are exported. Subscribers i and j indicates the sector and the country, respectively.

Equations (3) and (4) represents the equilibrium conditions for the production of domestic goods and the production of imported goods, respectively. Equation (5) is the equilibrium condition that adds a constraint in the input-output coefficients. The sum of the elements on the rows of the sector i in equation (3) must be equal to the sum of the sales for all domestic and intermediate use in the economy for this same sector i.

Similarly, in equation (4), the sum of the column elements j indicate the total imports of the sector i, which must be equal to the sum of the sales of the product of country j in the same sector for all users of the economy, including intermediate inputs for all sectors, final household consumption and gross fixed capital formation. Finally, the elements of equation (5) imply that the total production, X, in each sector i must be equal to the sum of the value added directly in the sector i and equal to the cost of intermediate inputs for all domestic and imported production.

In fact, the market potential for non-Brexit cases will be expressed as the sum of the technical production coefficients for all i sectors of j countries of the European Union. In the case of Brexit, the market potential will be measured by means of the hypothetical extraction of the United Kingdom from the European Union [6]. By doing so,

[6] about hypothetical extraction see Dietzenbacher, Linden e Steenge (1993) and Perobelli et al. (2006)

the United Kingdom's market potential will be the sum of the United Kingdom's own technical production coefficients. The market potential of the Rest of the European Union will be the sum of the technical coefficients of all i sectors of j countries of the European Union.

2.3 Productive Integration

To measure the degree of productive integration, defined as the degree of productive interdependence among i sectors of j countries, we made use of the value-added indicator on gross exports (VAX rate), which is a traditional measure of the Global Value Chains. The higher the coefficient, the lower the degree of productive integration of the country in this segment (JOHNSON; NOGUERA, 2012; HUMMELS; ISHII; YI, 2001).

Following Johnson e Noguera (2012), Koopman, Wang e Wei (2012), Timmer et al. (2015), Baldwin e Lopez-Gonzalez (2015), we can obtain the domestic content from an interregional model of input-output, as shown in equations (3) a (5), by the following expression:

$$VAX = A_v'(I-A)^{-1}F^{DM} \tag{6}$$

where A_v' represents the row vector A_v transposed share of total value added for the sector i from the country j, of size $1 \times n$, $(I-A)^{-1}$ is the inverse of Leontief and F^{DM} is a column vector of final demand for domestic D and imported M products.

Each element of the column vector of equation (6), with size $n \times 1$, can be interpreted as the externally share of value added in the production of exported domestic goods. According to Hummels, Ishii e Yi (2001), Koopman, Wang e Wei (2012), it can be considered as a measure of productive integration.

2.4 Labor Cost

Firms consider tha the cost of the labor force is a key variable for the location decision process (THUNEN, 1966; LAUNHARDT, 1885; DUNN, 1954; SCHWEIZER; VARAIYA, 1976; BECKMANN, 1972). Therefore, this variable should reflect the costs absorbed by firms taking into account a skilled $\Psi1$ and unskilled $\Psi2$ labor force for the same amount of production X in each sector i and in each country j.

The cost of the high skilled labor and low skilled labor can be obtained by the product between the participation of each type of labor in the total remunerations [7] and the gross value of the production of each sector i and for each country j, X_{ij}.

[7] For further details on the construction of labor force participation in total remuneration see Dietzenbacher et al. (2013)

2.5 Displacement Costs

The displacement costs impact the relocation decision of firms (ISARD, 1956; THÜNEN, 1875; LAUNHARDT, 1885). In this sense, the higher the displacement costs, the lower the firm's i availability to locate in another j country.

As an result, the displacement costs should mimic the firm's ability to obtain productive advantages associated with its relocation in the new region. For the purposes of this paper, displacement costs are associated with differences in the share of high skilled labor and low skilled labor between different countries for the same quantity produced:

$$\mu = (\Psi 1_{ij} - \Psi 2_{ij}) X_{ij}, \forall s \tag{7}$$

$$= (\Psi 1_{ij} - \Psi 2_{ij}) X_{ij}, \forall (1 - s) \tag{8}$$

where $\Psi 1_{ij}$ represents the share of the remuneration of high skilled labor over total remuneration in the sector i of country j, $\Psi 2_{ij}$ represents the share of the remuneration of low skilled labor over total remuneration in the sector i of country j and X_{ij} is the total production in each sector i of country j.

The intuition is that when the participation of the high skilled labor in the sector i of country j is higher than the participation of the low skilled labor, the cost of displacement is positive for the same amount of production X_{ij}. It indicates that the cost will be the same per unit of production, but with more participation of the high skilled labor, which may represent, for example, productivity gains (BALDWIN; LOPEZ-GONZALEZ, 2015; GANDOLFO, 1998; HUMMELS; ISHII; YI, 2001).

3 The Evolutionary Brexit Game Model

In an evolutionary game it is assumed bounded rationality, a large population, n, of players ($n \to \infty$) and an implicit recognition that agents learn. Every period, a player is randomly matched with another player and they play a two-player game. Each agent is randomly assigned a strategy at the initial step ($t = 0$), which can be updated over time via the systematic interaction with other agents. Thus, one player can imitate other players' strategies.

Following this intuition, now we intend to evaluate how robust is the strategic behavior of firms that will decide whether to locate in the United Kingdom (UK) or the rest of the European Union (EU). Therefore, the strategies available to each player are (UK) and (EU). To achieve this goal, consider two representative firms tagged as firm A and firm B. Firm A belongs to a population of firms that have a manufacturing plant in one of the European Union member countries, excluding the United Kingdom (UK). Firm B belongs to a population of firms that have a production plant in the UK.

At each interaction, a firm tagged by type A randomly competes with a firm tagged by type B. According to the evolutionary game theory approach, this situation characterizes a two-dimensional[8] game, in which two populations of firms compete against each other. Players are invited to play multiple times and do not compete against their peers. In this sense, there will only be competition among rivals' population of firms. For simplicity, the intra-population's competition is not considered.

The two population of firms (Type A; Type B) evaluate the possibility of relocation according to the probability of occurrence (s) or not ($1-s$) of Brexit. Let the row player be one representative firm of type A and the column player be one representative firm of type B. The payoff matrix of the stage game is given by (9):

$$\begin{array}{c} \\ UK \\ EU \end{array} \begin{array}{c} \quad UK \qquad\qquad EU \\ \begin{pmatrix} \pi_{A,1}\,;\,\pi_{B,1} & \pi^{'}_{A,1}\,;\,\pi^{'}_{B,2} \\ \pi^{'}_{A,2}\,;\,\pi^{'}_{B,1} & \pi_{A,2}\,;\,\pi_{B,2} \end{pmatrix} \end{array} \tag{9}$$

The factors described in the previous section are contemplated in the game payoff matrix. Thus, given the occurrence of Brexit with probability s, the market potential is given by (ρ). The measure of the productive integration is provided by (θ). The factors (ψ_1) and (ψ_2) measures the cost of skilled and unskilled labor, respectively. The displacement cost is given by (μ). The weight assigned to each of the factors, conditioned to the occurrence of Brexit with probability s is given by w_i, where $i = \{\rho, \theta, \psi, \mu\}$ and $\Sigma w_i = 1$.

For the case in which there is no occurrence of brexit, with probability $1-s$, the factor that measures the market potential is given by ($\mathbf{P}$). The productive integration is (Θ). The factors (Ψ_1) and (Ψ_2) measures the costs of the label force and the displacement cost is given by ($\mathbf{M}$). The weight assigned to each of the factors, conditioned to the non-occurrence of Brexit with probability $(1-s)$ is given by W_I, where $I = \{\mathbf{P}, \Theta, \Psi, \mathbf{M}\}$ and $\Sigma W_I = 1$.

Thus, the payoffs in (9) are expressed as follows:

$$\begin{aligned} \pi_{A,1} = s[1/2\, w_\rho \rho_{A,1} + 1/2\, w_\theta \theta_{A,1} + 1/2\, w_\psi\, (\psi_{1,A1} + \psi_{2,A1}) + w_\mu \mu_{A,1}] + \\ (1-s)\, [1/2\, W_{\mathbf{P}} \mathbf{P}_{A,1} + 1/2\, W_\Theta \Theta_{A,1} + 1/2\, W_\Psi\, (\Psi_{1,A1} + \Psi_{2,A1}) + W_{\mathbf{M}} \mathbf{M}_{A,1}] \end{aligned} \tag{10}$$

$$\begin{aligned} \pi_{B,1} = s[1/2\, w_\rho \rho_{A,1} + 1/2\, w_\theta \theta_{A,1} + 1/2\, w_\psi\, (\psi_{1,A1} + \psi_{2,A1})] + \\ (1-s)\, [1/2\, W_{\mathbf{P}} \mathbf{P}_{A,1} + 1/2\, W_\Theta \Theta_{A,1} + 1/2\, W_\Psi\, (\Psi_{1,A1} + \Psi_{2,A1})] \end{aligned} \tag{11}$$

[8] Friedman (1991) for a detailed exposition and explanation.

$$\pi'_{A,1} = s[w_\rho \rho_{A,1} + w_\theta \theta_{A,1} + 1/2\, w_\psi\, (\psi_{1,A1} + \psi_{2,A1}) + w_\mu \mu_{A,1}] + \\ (1-s)\, [W_{\mathbf{P}} \mathbf{P}_{A,1} + W_\Theta \Theta_{A,1} + 1/2\, W_\Psi\, (\Psi_{1,A1} + \Psi_{2,A1}) + W_{\mathbf{M}} \mathbf{M}_{A,1}] \quad (12)$$

$$\pi'_{B,2} = s[w_\rho \rho_{A,2} + w_\theta \theta_{A,2} + 1/2\, w_\psi\, (\psi_{1,A2} + \psi_{2,A2}) + w_\mu \mu_{A,2}] + \\ (1-s)\, [W_{\mathbf{P}} \mathbf{P}_{A,2} + W_\Theta \Theta_{A,2} + 1/2\, W_\Psi\, (\Psi_{1,A2} + \Psi_{2,A2}) + W_{\mathbf{M}} \mathbf{M}_{A,2}] \quad (13)$$

$$\pi'_{A,2} = s[w_\rho \rho_{A,2} + w_\theta \theta_{A,2} + 1/2\, w_\psi\, (\psi_{1,A2} + \psi_{2,A2})] + \\ (1-s)\, [W_{\mathbf{P}} \mathbf{P}_{A,2} + W_\Theta \Theta_{A,2} + 1/2\, W_\Psi\, (\Psi_{1,A2} + \Psi_{2,A2})] \quad (14)$$

$$\pi'_{B,1} = s[w_\rho \rho_{A,1} + w_\theta \theta_{A,1} + 1/2\, w_\psi\, (\psi_{1,A1} + \psi_{2,A1})] + \\ (1-s)\, [W_{\mathbf{P}} \mathbf{P}_{A,1} + W_\Theta \Theta_{A,1} + 1/2\, W_\Psi\, (\Psi_{1,A1} + \Psi_{2,A1})] \quad (15)$$

$$\pi_{A,2} = s[1/2\, w_\rho \rho_{A,2} + 1/2\, w_\theta \theta_{A,2} + 1/2\, w_\psi\, (\psi_{1,A2} + \psi_{2,A2})] + \\ (1-s)\, [1/2\, W_{\mathbf{P}} \mathbf{P}_{A,2} + 1/2\, W_\Theta \Theta_{A,2} + 1/2\, W_\Psi\, (\Psi_{1,A2} + \Psi_{2,A2})] \quad (16)$$

$$\pi_{B,2} = s[1/2\, w_\rho \rho_{A,2} + 1/2\, w_\theta \theta_{A,2} + 1/2\, w_\psi\, (\psi_{1,A2} + \psi_{2,A2}) + w_\mu \mu_{A,2}] + \\ (1-s)\, [1/2\, W_{\mathbf{P}} \mathbf{P}_{A,2} + 1/2\, W_\Theta \Theta_{A,2} + 1/2\, W_\Psi\, (\Psi_{1,A2} + \Psi_{2,A2})\, W_{\mathbf{M}} \mathbf{M}_{A,2}] \quad (17)$$

For simplicity, the payoff matrix presented in (9) is normalized. This procedure will not affect the best response structure of the game and will simplify the analysis. Assume that $\eta_A = (\pi_{A,1} - \pi'_{A,2})$, $\eta_B = (\pi_{B,1} - \pi'_{B,2})$, $\upsilon_A = (\pi_{A,2} - \pi'_{A,1})$, and $\upsilon_B = (\pi_{B,2} - \pi'_{B,1})$. Players are randomly matched and compete against each other in a one-shot game. The level of aggregate strategies of populations do not change all at once. In fact, they continuously update their strategic behavior over time. The matrix (18) is the basis of the evolutionary dynamics of the game.

$$\begin{array}{cc} & \begin{array}{cc} UK & EU \end{array} \\ \begin{array}{c} UK \\ EU \end{array} & \begin{pmatrix} \eta_A; \eta_B & 0; 0 \\ 0; 0 & \upsilon_A; \upsilon_B \end{pmatrix} \end{array} \quad (18)$$

Let f_i be the fraction of firms using strategy UK in population i, so the fraction of firms adopting EU in population i is $1 - f_i, i = A, B$. The observed variations in the proportion of players who adopt each of the strategies reflects their evolutionary process within each population. Note that this relative frequency can be understood as the probability that a player will play a given strategy. Both the evolution of the game and the strategic behavior of the firms is conditioned to the fitness of their strategies.

The fitness, according to Binmore (1992) and Samuelson (2002), depends on the player's payoff for a given strategy and on the relative frequency of the strategies observed in both populations. That is, players make decisions based on the expected utility of their payoffs. As we will see in the results section, it is possible that from the normalized matrix (18) we have a situation in which more than one Nash equilibrium compose the solution set. Therefore, the following question arises: departing from an initial game condition, which equilibrium will be reached?

Without loss of generality, to answer this question, we first analyze the evolution and the robustness of firm's strategic behavior with the analytic solution of the replicator dynamics (RD) system (TAYLOR; JONKER, 1978), which is a very general ordinary differential equation (ODE) system in evolutionary game theory. As presented in Hirth (2014), in a dynamic system, the growth rate $\dot{f}_A/f_A$ equals the strategy UK's fitness $e^1A(f_B, 1-f_B)^T$ less the average fitness $(f_A, 1-f_A)A(f_B, 1-f_B)^T$ of population A, where $\dot{f}_A = df_A/dt$ and $e^1 = (1, 0)$ represents that all firms from population A chooses strategy UK.

Let A $= \begin{bmatrix} \eta_A & 0 \\ 0 & \upsilon_A \end{bmatrix}$ be the payoff matrix of a representative firm A. After some trivial matrix algebra, the replicator dynamics equation for population A is $\dot{f}_A = f_A((e^1 - (f_A, 1-f_A))A(f_B, 1-f_B)^T$. Substituting the values of A (payoffs earned by the representative firm A), in order to derive the replicator dynamics system for population A and B:

$$\dot{f}_A = f_A(1-f_A)[\eta_A f_B - \upsilon_A(1-f_B)] \tag{19}$$

$$\dot{f}_B = f_B(1-f_B)[\eta_B f_A - \upsilon_B(1-f_A)] \tag{20}$$

$\dot{f}_A$ and $\dot{f}_B$ represent the growth rate of the proportion of firms that adopt the first pure strategy P within each population (A and B). The stability of the system is a coordinate $(f_A, f_B) \in [0,1] \times [0,1]$, in which $\dot{f}_A = \dot{f}_B = 0$ is a necessary condition for the stationarity of (19) and (20). To check the stability of the points candidates for an ESS, i.e., an asymptotically stable steady state for the two-population game, we must use the Jacobian matrix (Ω). Calculating the eigenvalues: $\Omega(f_A, f_B) = \begin{bmatrix} \frac{d\dot{f}_A}{df_A} & \frac{d\dot{f}_A}{df_B} \\ \frac{d\dot{f}_B}{df_A} & \frac{d\dot{f}_B}{df_B} \end{bmatrix}$; doing the determinant $det(\Omega - \lambda_j) = \begin{vmatrix} \frac{d\dot{f}_A}{df_A} - \lambda & \frac{d\dot{f}_A}{df_B} \\ \frac{d\dot{f}_B}{df_A} & \frac{d\dot{f}_B}{df_B} - \lambda \end{vmatrix} = 0$. We finally have that $\lambda_{1,2} = tr\Omega \pm \sqrt{tr\Omega^2 - 4det\Omega}$.

For the stationary point to be asymptotically stable, the eigenvalues $\lambda_{1,2}$ of the matrix (Ω) evaluated at points that hold the condition $\dot{f}_A = 0$ and $\dot{f}_B = 0$ must have negative real parts. The analytical solution and the phase diagrams, responsible for the evolutionary game dynamics, were made with the open source software Dynamo[9]

4 Results

To generate the following results, we assign weights to the variables obtained from the input-output[10] analysis for sectors 1 and 41. Table (2) summarizes the values chosen for each of the evaluated cases.

Homogeneous Scenario		Scenario 1		Scenario 2		Scenario 3	
(s = 0.5)	(1-s = 0.5)	(s)	(1-s)	(s)	(1-s)	(s)	(1-s)
$w_\rho = 0.25$	$W_{\mathbf{P}} = 0.25$	$w_\rho = 0.4$	$W_{\mathbf{P}} = 0.1$	$w_\rho = 0.6$	$W_{\mathbf{P}} = 0.05$	$w_\rho = 0.3$	$W_{\mathbf{P}} = 0.05$
$w_\theta = 0.25$	$W_\Theta = 0.25$	$w_\theta = 0.1$	$W_\Theta = 0.1$	$w_\theta = 0.05$	$W_\Theta = 0.05$	$w_\theta = 0.05$	$W_\Theta = 0.05$
$w_\psi = 0.25$	$W_\Psi = 0.25$	$w_\psi = 0.1$	$W_\Psi = 0.4$	$w_\psi = 0.05$	$W_\Psi = 0.6$	$w_\psi = 0.05$	$W_\Psi = 0.3$
$w_\mu = 0.25$	$W_{\mathbf{M}} = 0.25$	$w_\mu = 0.4$	$W_{\mathbf{M}} = 0.4$	$w_\mu = 0.3$	$W_{\mathbf{M}} = 0.3$	$w_\mu = 0.6$	$W_{\mathbf{M}} = 0.6$

Table 1 – Summary of the weighted factors considered in each Scenario.

Case I can be seen as a homogeneous Scenario, that is, the firms consider the probability of occurrence of Brexit given by $s = 1/2$ and also distribute homogeneous weights for each of the factors considered in the analysis.

In case II, it is considered a high probability ($s = 0.7$) of occurrence of Brexit. This case still presents three different Scenarios, in which the firm can order its preference for the considered factors, assigning greater weights to those variables potentially more relevant for its decision making. The case

Case III refers to a low probability of occurrence of Brexit ($s = 0.3$). In the same way as in Case II, firms consider three different Scenarios, in which they list the variables that would be most relevant in the localization decision process.

4.1 Case I

4.1.1 The Homogeneous Scenario

Sector 1

$$\begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.1; -0.1 & \mathbf{0;0} \\ \mathbf{0;0} & -0.34; -0.41 \end{pmatrix}$$

Sector 41

$$\begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.52; -0.35 & 0; 0 \\ \mathbf{0;0} & 0.11; -0.05 \end{pmatrix}$$

[9] See Sandholm, Dokumaci e Franchetti (2012).

[10] In the appendix, the table (ref: A1) presents the values of each of the parameters referring to sectors 1 and 41, respectively. These values were extracted from the input-output matrix. Departing from the gross values, we performed a comparative static exercise, in which the values were scaled for the interval $(0, 1)$. The largest parameters observed assumed value equal to 1.

The homogeneous Scenario characterizes an anti-coordination game for firms operating in Sector 1. In the phase diagrams below, black dots correspond to ESS strategies and white dots are unstable. In this way, let Φ^{ESS} be the set of evolutionarily stable strategies.

For the firms that operate in Sector 1, we have $\Phi^{ESS} = \{(EU, UK); (UK, EU)\}$. For the Sector 41, we have $\Phi^{ESS} = \{(EU, UK)\}$. The colors of the phase diagram can be interpreted as follows: the region in red corresponds to the maximum speed of convergence of the system. In blue, we have the region where the system slowly approaches stationary states.

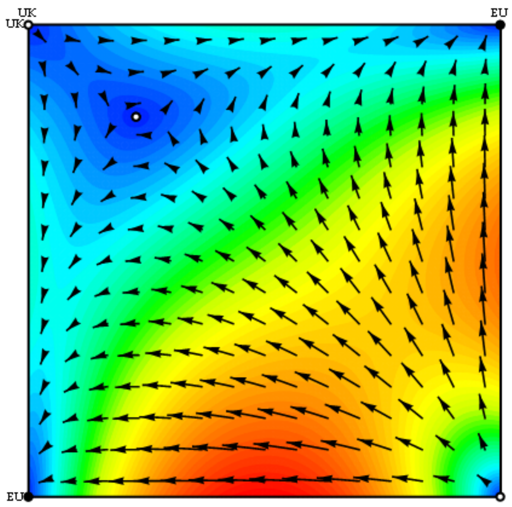

Figure 1 – Phase diagram for Sector 1

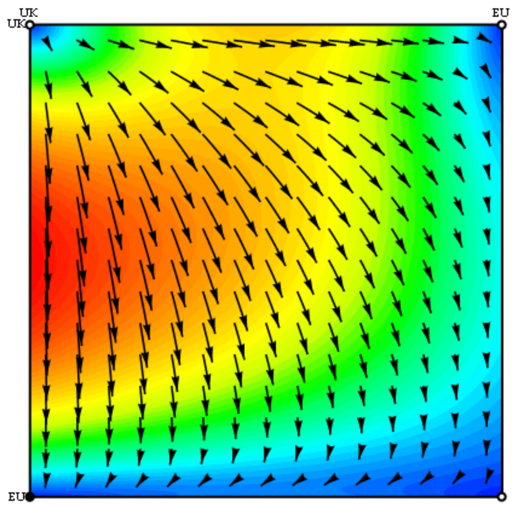

Figure 2 – Phase diagram for Sector 41

For Sector 1, from the figure 1, we see that when the proportion of firms adopting the pure strategy (UK) in both populations at the initial time of the game is high, standing in the neighborhood of the unstable point inside the diagram, the ESS is given by $\Phi^{ESS} = (UK; EU)$. In words, this means that firms that have a production plant in the EU will be relocated to the UK. On the other hand, firms that operate in the UK, observing this movement, will do the opposite way, that is, they will be located in the EU.

The economic intuition of this equilibrium is that, given the weights assigned to the parameters, the firm has a benefit when it incurs at the displacement cost, since it relocates in an unsaturated market. When we assume as initial condition that the firms of both populations are willing to locate in UK, following the same economic intuition, we see that the ESS is given by $\Phi^{ESS} = (EU; UK)$.

Therefore, we observed that this result may reflect the characteristics of Sector 1, since the cost of labor and the cost of displacement in this sector is relatively low. Another argument may be due to the low differentiation of the products associated to Sector 1, which tends to cause firms not to look for saturated markets.

For Sector 41, whose characteristics are quite distinct, we observe in figure 2 that the unique ESS is given by $\Phi^{ESS} = (EU, UK)$. The economic intuition of this result reflects the high costs associated with the workforce and, consequently, the high costs of displacement. So, under these circumstances, the best response for a firm that is operating in the EU is not to move to the UK. The rationale is analogous for firms operating in the UK.

4.2 Case II

In Case II we assign a high probability $(s = 0.7)$ of Brexit occurrence. In this way, it is intended to capture the strategic behavior of firms facing such possibility, as well as the degree of influence of each of the factors in the equilibrium condition of the game. By doing so, we consider the following three possible Scenarios.

4.2.1 Scenario I: emphasis on ρ, μ and $\mathbf{M}$

Sector 1

$$\begin{array}{c} \\ UK \\ EU \end{array}\begin{pmatrix} UK & EU \\ \mathbf{0.03;0.13} & 0;0 \\ 0;0 & -0.31;-0.48 \end{pmatrix}$$

Sector 41

$$\begin{array}{c} \\ UK \\ EU \end{array}\begin{pmatrix} UK & EU \\ -0.71;-0.43 & 0;0 \\ 0;0 & \mathbf{0.44;0.19} \end{pmatrix}$$

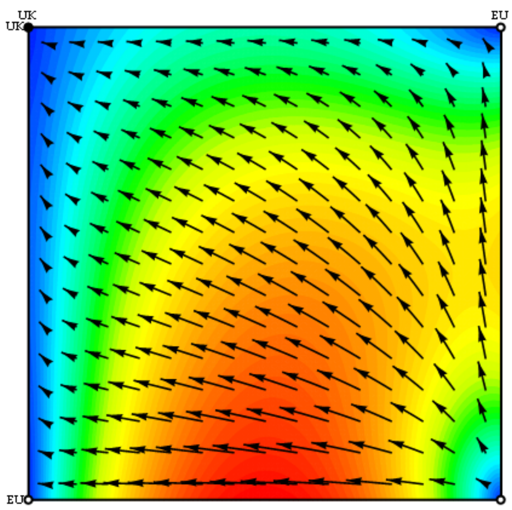

Figure 3 – Phase diagram for Sector 1

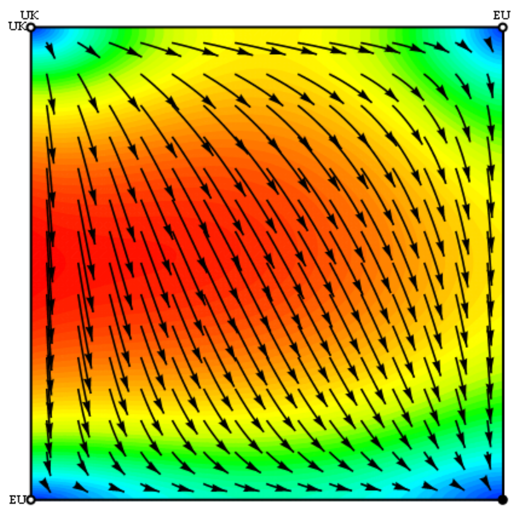

Figure 4 – Phase diagram for Sector 41

For Sector 1, according to figure 3, we see that there is a balance in dominant strategy. That is, regardless of what the rival firm adopts as strategy, the best response is always to locate in UK. This equilibrium contrasts with that observed in the previous case, because facing a high probability of occurrence of Brexit, firms initially located in the EU will migrate to the UK and will share the market potential of this region.

One possible explanation can be attributed to less productive integration as a consequence of the UK's exit from the EU common market. This would reduce the level of international competition, making the UK market attractive to firms operating in Sector 1. Another possible exogenous explanation to our model, is the fact that this industry is extreme dependent on natural resources and high subsidies.

For Sector 41, from figure 4 we also observe a game with dominant strategies, but the movement of firms is towards the EU. That is, firms initially located in the UK will choose to incur in displacement costs because they consider market potential as a relevant variable in the criterion of location decision. In this way, the benefits acquired from this relocation outweigh the costs associated with this movement.

4.2.2 Scenario II: emphasis on ρ and Ψ

Sector 1

$$\begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.18;-0.12 & \mathbf{0;0} \\ \mathbf{0;0} & -0.10;-0.31 \end{pmatrix}$$

Sector 41

$$\begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.66;-0.45 & 0;0 \\ 0;0 & \mathbf{0.37;0.12} \end{pmatrix}$$

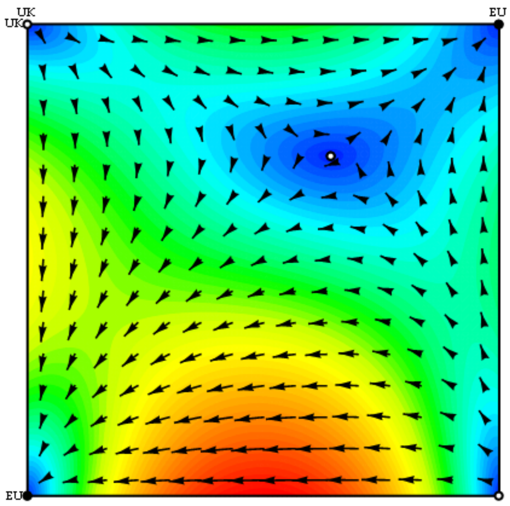

Figure 5 – Phase diagram for Sector 1

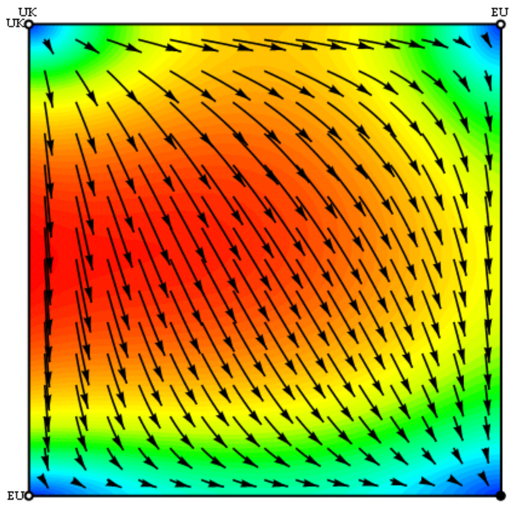

Figure 6 – Phase diagram for Sector 41

In this Scenario, we consider that the firms attribute greater weight to the market potential in view of a high probability of occurrence of Brexit. On the other hand, if it does not occur, firms attach greater weight to the cost of labor.

For Sector 1, the economic intuition behind this particular case follows the argument given to the equilibrium found in case I (see figure 1). In this way, we can emphasize the inherent characteristics of the sector as determinants of the equilibrium condition reached by the analysis of the phase diagram of the figure 5. Following this argument, we can explain the equilibrium observed in Sector 41, provided by the figure 6. That is, firms will choose to split the EU market.

4.2.3 Scenario III: emphasis on μ and $\mathbf{M}$

In this Scenario, we consider that firms attribute greater weight to the displacement cost in view of a high or low probability of occurrence of Brexit. Thus, by figure 7 we observe that, for Sector 1, there is a balance in dominant strategies given by (UK, UK). Although firms place more weight on the displacement cost, they observe greater benefits

$$\text{Sector 1} \qquad \begin{matrix} UK \\ EU \end{matrix} \begin{pmatrix} UK & EU \\ \mathbf{0.34;0.47} & 0;0 \\ 0;0 & -0.52;-0.67 \end{pmatrix} \qquad \text{Sector 41} \qquad \begin{matrix} UK \\ EU \end{matrix} \begin{pmatrix} UK & EU \\ -0.90;-0.48 & 0;0 \\ 0;0 & \mathbf{0.72;0.37} \end{pmatrix}$$

by sharing the UK market. The economic intuition is similar to that attributed to Scenario I, being related to the market potential observed in the UK. The rationale is analogous for Sector 41, but the equilibrium in dominant strategies is formed by both firms sharing the EU market, as we can see by figure 8.

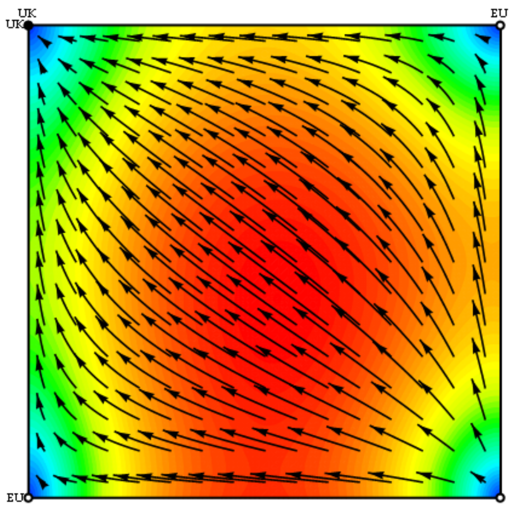

Figure 7 – Phase diagram for Sector 1

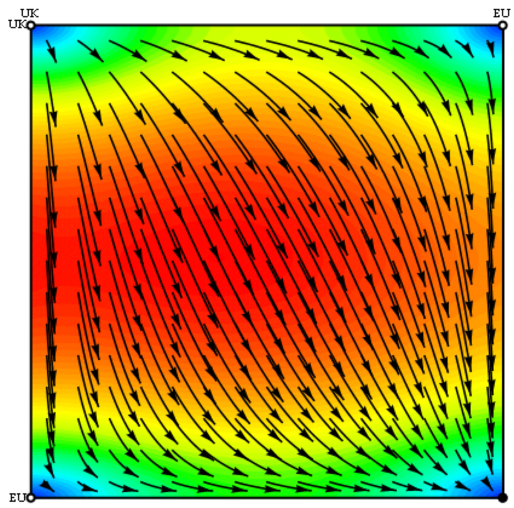

Figure 8 – Phase diagram for Sector 41

4.3 Case III

In Case III we assign a low probability ($s = 0.3$) of Brexit occurrence. In this way, it is intended to capture the strategic behavior of firms facing such possibility, as well as the degree of influence of each of the factors in the equilibrium condition of the game. By doing so, we consider the following three possible Scenarios.

4.3.1 Scenario I: emphasis on ρ, μ and $\mathbf{M}$

$$\text{Sector 1} \qquad \begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ \mathbf{0.15;0.06} & 0;0 \\ 0;0 & -0.38;-0.47 \end{pmatrix} \qquad \text{Sector 41} \qquad \begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.41;0.06 & 0;0 \\ \mathbf{0;0} & 0.18;-0.21 \end{pmatrix}$$

For a better understanding of the results presented in this specific case, we invite the reader to compare this outocome with that observed in Scenario I of case II, in which the equilibrium dynamics could be seen by figures 3 and 4, respectively. Note that even when we assign a lower likelihood of occurrence for Brexit, the strategic behavior of firms operating in Sector 1 is not different from that observed when Brexit's probability of occurrence is high.

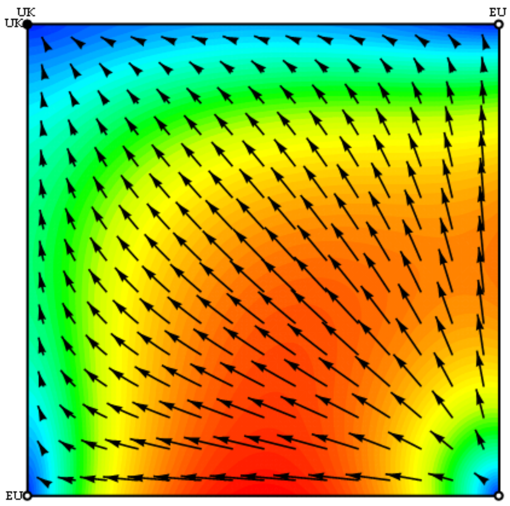

Figure 9 – Phase diagram for Sector 1

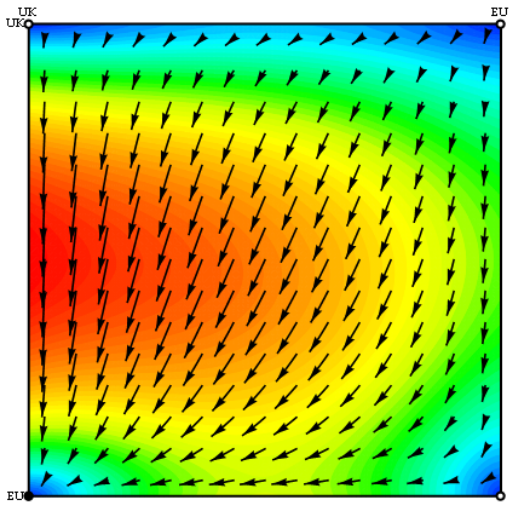

Figure 10 – Phase diagram for Sector 41

Therefore, the balance observed in figure 9 shows a possible UK locational advantage in the production of goods associated with the Sector 1. However, for Sector 41, this logic does not hold. Specially when we compare the equilibrium dynamics observed in figure 10 with that one observed in figure 4. In other words, it is possible to conclude that, if we consider an unlikely exit of the United Kingdom from the European Union's common market, together with a greater relevance to the displacement costs, one may observe that firms prefer not to pay these costs. Thus, they will remain located in their respective countries of origin. Intuitively, this could mean that, given the weights considered, the location decision becomes more influenced by the probability associated with Brexits occurrence or not.

4.3.2 Scenario II: emphasis on ρ and Ψ

$$\text{Sector 1} \qquad \begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.01; -0.05 & \mathbf{0;0} \\ 0; 0 & -0.19; -0.47 \end{pmatrix} \qquad \text{Sector 41} \qquad \begin{array}{c} \\ UK \\ EU \end{array} \begin{pmatrix} UK & EU \\ -0.21; 0.14 & 0; 0 \\ \mathbf{0;0} & 0.03; -0.31 \end{pmatrix}$$

For Sector 1, the equilibrium dynamics observed in figure 11 suggests that firms will locate in unsaturated markets. Note that, in this Scenario, a greater weight is attributed to labor costs if the Brexit event does not occur. Therefore, it is possible to infer that the expected cost with the relocation is lower than the expected cost with the labor force. This logic is inverse as we observe the equilibrium dynamics of the figure 12 for Sector 41. Firms choose to locate in their home countries, since the expected value of the cost of displacement is greater than the expected value of labor cost.

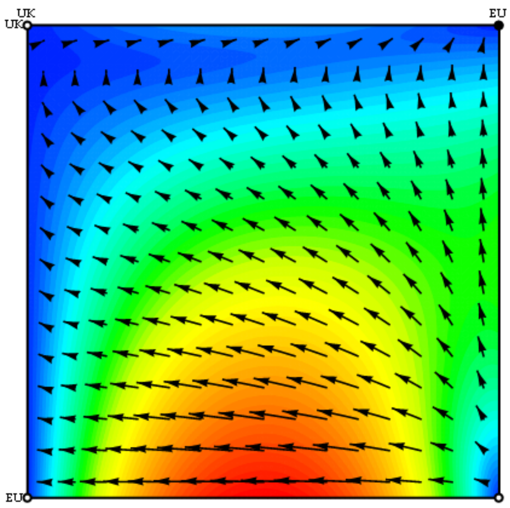

Figure 11 – Phase diagram for Sector 1

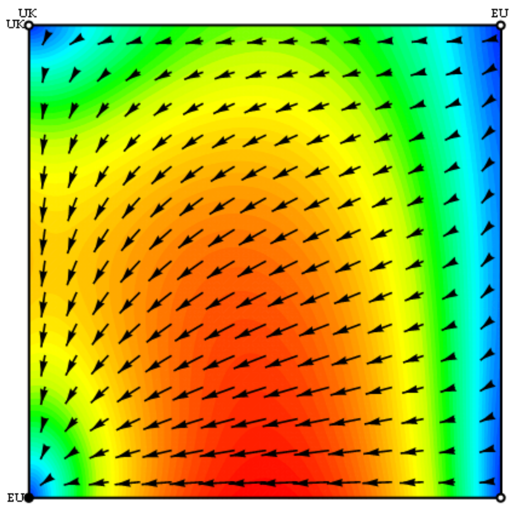

Figure 12 – Phase diagram for Sector 41

Therefore, comparing this result with that obtained in Scenario II of case II, which is evidenced by figure 6, we have another indication that the uncertainty associated with Brexit may exert a greater influence on the firm location decision than the weights attributed to each of the factors considered.

4.3.3 Scenario III: emphasis on μ and $\mathbf{M}$

$$\begin{array}{cc} \text{Sector 1} & \text{Sector 41} \\ \begin{array}{c} \\ UK \\ EU \end{array}\begin{pmatrix} UK & EU \\ \mathbf{0.44;0.32} & 0;0 \\ 0;0 & -0.56;-0.49 \end{pmatrix} & \begin{array}{c} \\ UK \\ EU \end{array}\begin{pmatrix} UK & EU \\ -0.62;0.09 & 0;0 \\ \mathbf{0;0} & 0.49;-0.05 \end{pmatrix} \end{array}$$

In this last Scenario, we attribute greater relevance to the cost of displacement in view of Brexit's possibility of occurrence or not. What is observed for Sector 1 is a game in dominant strategies, whose dynamics is given by the figure 13. Note that both firms will compete in the UK market.

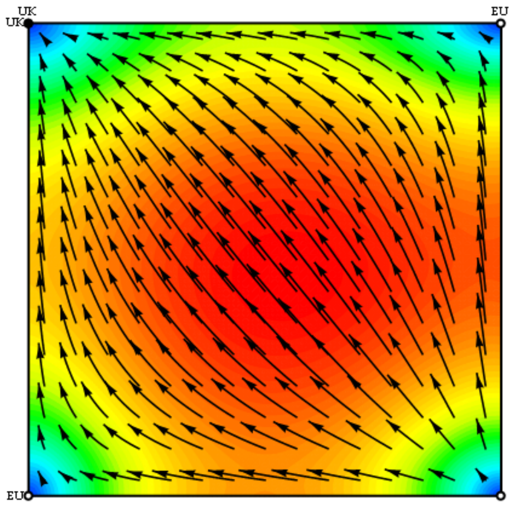

Figure 13 – Phase diagram for Sector 1

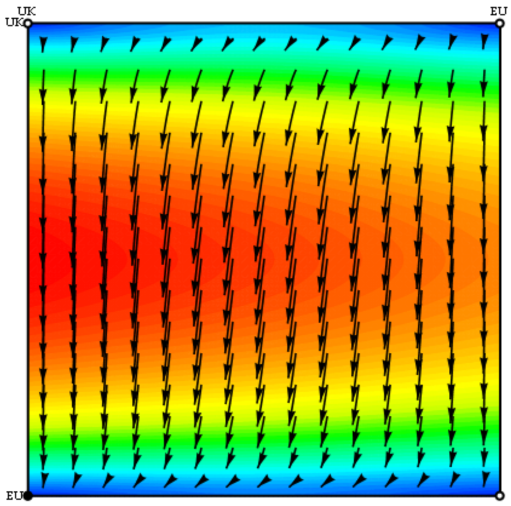

Figure 14 – Phase diagram for Sector 41

This result suggests a possible UK locational advantage, since the expected benefits of the combination involving market potential, labor costs, and productive integration outweigh the costs associated with firm A's (which is originally located in the EU) displacement. In addition, the possible occurrence of Brexit, for this sector, does not seem to be preponderant for the firm locating decision, since the equilibrium is the same as in the case in which the probability of occurrence of Brexit is high.

On the other hand, for Sector 41, it is possible to evaluate that the probability associated to the occurrence of Brexit is preponderant for the balance of the game. Note that the result presented in this case is opposite to that observed in Scenario III of case

II, where we attribute a high probability to the Brexit event. The equilibrium dynamics observed in figure14 suggests that, given a low probability of Brexit, the best strategy for both firms is to stay in their home countries, that is, the ESS is given by (EU, UK).

5 Conclusion

The objective of this article was to analyze the influence of Brexit on firm's location decision. In order to achieve this goal, two sectors with very particular characteristics were chosen. Sector 1, Crop and animal production, hunting and related service activities, characterized by high locational advantages, strong dependence on natural resources and largely subsidized by local governments.

On the other hand, the Sector 41, Financial services activities, except insurance and pension funding, characterized by being a service provider, with high integration and very important in the structure of the United Kingdom economy.

The results indicated that the occurrence of Brexit can be determinant in the decision of firm location, although the sensitivity of the firms to Brexit is conditioned to the sector of performance and the relevance given to each of the factors in the decision process.

In this sense, the results obtained suggest that, in sectors associated with high locational advantages, strong dependence on natural resources and traditionally benefited by fiscal subsidies, the prospect of Brexit indicates that in an environment of uncertainty, firms tend to search for unsaturated markets.

By looking into another direction, firms associated with high productive integration in the services sector appear to be more sensitive to the uncertainties brought by Brexit. In this sense, the greater the prospect of Brexit, the greater the probability of firms in this sector to move towards the European Union.

There are many possible direction to be taken in extensions to this paper. One could analyze the location decision process of firms under uncertainty by considering a greater degree of disaggregation of countries and other productive factors. We could also direct the analysis of the location decision to other sectors and with alternative methodologies, such as the Agent Based Models. By using this simulation modelling, it is possible to establish a local neighborhood criterion in which the agents interact and develop a learning capacity.

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6 Appendix

Sector 1 - Crop and animal production, hunting and related service activities											
		Parameters (s)						**Parameters(s)**			
	ρ	θ	$\psi1$	$\psi2$	μ		ρ	θ	$\psi1$	$\psi2$	μ
UK	21,73892	0,941718	10489,74876	11224,4568	4587,90638	UK	0,04	1	1	0,51	1
EU	586,3658	0,932038	9166,843022	22044,2141	-7126,015	EU	1	0,99	0,87	1	-1,55
		Parameters (1-s)						**Parameters (1-s)**			
		Θ	$\Psi1$	$\Psi2$				Θ	$\Psi1$	$\Psi2$	
UK	608,10	0,93	12151,31	13002,39	5314,62	UK	1	1	1	0,52	1
EU	608,10	0,93	10322,48	24823,26	-2546,45	EU	1	1	0,85	1	-0,48
Sector 41 - Financial services activities, except insurance and pension funding											
		Parameters (s)						**Parameters (s)**			
	ρ	θ	$\psi1$	$\psi2$	μ		ρ	θ	$\psi1$	$\psi2$	μ
UK	608,10	0,96	121413,93	15362,86	-7593,70	UK	1,00	1,00	1,00	1,00	-1,00
EU	608,10	0,96	33005,67	2466,81	-4184,23	EU	1,00	1,00	0,27	0,16	-0,55
		Parameters (1-s)						**Parameters (1-s)**			
		Θ	$\Psi1$	$\Psi2$				Θ	$\Psi1$	$\Psi2$	
UK	21,74	0,94	34445,60	4358,50	-2154,36	UK	0,04	0,98	1,00	1,00	-1,33
EU	586,37	0,96	27505,51	2055,74	1619,04	EU	1,00	1,00	0,80	0,47	1,00

Table 2 – Standardization of variables for Sectors 1 and 41