Optimal Composition of the Public Spending and Economic Growth*

Jose Angelo Divino† Daniel T. G. N. Maciel‡ Wilfredo Sosa§

18th March 2017

Abstract

The objective of this paper is to investigate the relationship among the size of the government, composition of public spending, and economic growth. We expand the theoretical model due to Devarajan et al (1996) by including technological progress in a more general constant elasticity of substitution (CES) production function. In addition, we use a balanced panel data for the Brazilian states to estimate the model's structural parameters and compute optimal ratios derived from the theoretical modeling. We find that private capital has a higher share than government spending in the production. The estimated tax burden is below the optimal level implied by the model. The public spending in investment is considerable lower than in costing, as in developing countries with low economic dynamism. Finally, it is possible to increase taxation and government spending without hurting economic growth of the Brazilian states.

Keywords: Public spending; Optimal taxation; Economic growth.
JEL Codes: O41, H50.

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*The authors thank CNPq for financial support.

†Corresponding author. Graduate Program of Economics, Catholic University of Brasilia, SGAN 916, Office A-118, Zip: 70790-000, Brasilia-DF, Brazil. E-mail: jangelo@pos.ucb.br.

‡Graduate Program of Economics, Catholic University of Brasilia. E-mail: daniel_mt_br@hotmail.com.

§Graduate Program of Economics, Catholic University of Brasilia. E-mail: sosa@ucb.br.

1 Introduction

Several countries around the world have recently faced episodes of fiscal crises due to the incapacity of their governments to bridge a deficit between public expenditures and tax revenues.[1] These crises share some common features, given that they are usually accompanied by economic, social, and political distresses and the recovery is painful to the society as hole because it simultaneously requires cuts in government expenditures and increases in taxes on individuals and firms. Given the relevance of the fiscal policy to a country's economic performance, it is important to keep an eye on both the relationship between the size of government and economic growth and the effects of the composition of the public expenditure on the country's growth rate. The latter issue rests on the fact that some public expenditures are seen as productive while others are considered unproductive in terms of their impacts on the economic activity. Thus, under this perspective, a country would be able to improve its economic performance by changing the mix between these two kinds of public expenditures.

The empirical and theoretical literatures have devoted a considerable amount of work to analyze the relationship among the size of the government, composition of the public expenditure, and economic growth. Aschauer (1989), Lindauer and Velenchik (1992), and Barro (1990, 1991), for instance, investigated the impacts of aggregate government spending on economic growth and productivity. In a pioneer study, Devarajan et al (1996) analyzed the relationship between composition of public expenditure and economic growth using both theoretical and empirical frameworks. Davoodi and Zou (1998), Xie et al (1999), and Zhang and Zou (1998) examined the growth effects of aggregate public expenditure by different levels of government in a fiscal-federalism environment. Finally, Zhang and Zou (2001) unified the previous literature by focusing on the growth impacts of the allocation of public expenditure among multiple sectors (such as health, education, transportation, among others) with multiple levels of government (such as local, state, and federal). It is still missing in the literature, however, studies on the optimal size of the government and on the optimal composition of the public expenditure.

For the Brazilian case, Rocha and Giuberti (2007) and Divino and Silva Jr. (2012) provide empirical evidence on the optimal composition of the

[1] For instance, the cases of Greece, Portugal, Italy, Spain, and more recently Brazil are well documented by the general media.

public expenditure for states and municipal districts, respectively. Both authors do not impose any a priori restriction on the productivity of the public expenditure and find that the optimal share of the current spending should range from 61 to 81% of the total public spending. However, no attempt is made by them to model the relationship among the optimal size and composition of the public expenditure and economic growth.

The objective of this paper is to fill this gap by investigating both the optimal size of the government in the economy and the optimal shares of productive and unproductive public expenditures in the aggregate government spending. To do so, we extend the framework proposed by Devarajan et al (1996) by including an exogenous technological progress in a more general constant elasticity of substitution (CES) function and showing how those optimal shares and government size depend on the structural parameters of the economy. We focus on economic growth instead of other measure of welfare for comparison purposes with the results by Devarajan et al (1996) and because it is important to identify the contribution of different components of the public expenditure to the economic growth.

Our major contribution is to show that the optimal size of the government in the economy, defined as the level of aggregate public expenditure over GDP that maximizes consumption growth, depends on the model's structural parameters. For the Brazilian states, the average optimal size is around 20% under a general parametrization of the model. Considering the special case of a Cobb-Douglas production function, that optimal size negatively depends on the share of the capital in the production function.

The optimal share of productive public expenditure relatively to the unproductive spending, on its turn, depends on the share of the productive spending and the elasticity of substitution between productive and unproductive public spending in the CES government production function. Considering the special case of a Cobb-Douglas function, that optimal share depends exclusively on the share of the productive expenditure on the aggregate government spending. Thus, it is crucial to have robust estimates of these parameters in order to calculate optimal ratios for any given country.

In the general case, however, the optimal taxation depends on the whole set of structural parameters. In particular, the technological progress has a direct effect on the optimal level of taxation while the share of private capital in total production has a negative effect on taxation. This theoretical finding coincides with the empirical results obtained for the Brazilian economy.

In order to estimate the structural parameters and find optimal ratios, we applied the model to a panel data for the Brazilian states in the recent period. A general CES production function, which combines private capital, composition of public expenditure, and technological progress, was estimated at the state level. We found the composition of public spending, level of taxation, and economic growth implied by the estimated structural parameters for the Brazilian states. Then, we computed the level of taxation and composition of the public expenditure that maximize the economic growth. We also performed a sensitivity analysis of the private capital productivity and average economic growth with respect to changes in the composition of public expenditure and in the total taxation.

The paper is organized as follows. The next section presents the model economy and derives the theoretical results. The empirical evidence for the Brazilian economy is reported and discussed in the third section. Finally, the fourth section is dedicated to the concluding remarks.

2 The model

The theoretical framework is based on Devarajan et al (1996), whose model is extended to consider a general CES (constant elasticity of substitution) functions for both government aggregate expenditure and economy production function under a minimal set of restrictions on the parameter values. In addition, we add technological progress to the aggregate CES production function. Thus, differently from the original model, our aggregate production function has three arguments, represented by private capital stock, k, aggregate government government spending, x, and exogenous technological progress, A.[2] They are combined into a CES function expressed as:

$$y_t = A_t \left[\alpha k_t^{-\zeta} + (1-\alpha) x_t^{-\zeta}\right]^{-\frac{1}{\zeta}} \tag{1}$$

with $1 \geq \alpha > 0,\ \zeta \in (-1,0) \cup (0,+\infty)$.

The aggregate government spending, x, is also given by a CES function which combines, say, productive, g_1, and unproductive, g_2. The government finances its spending by levying a flat-rate income tax, τ. The following equations expresses these relationships:

$$x_t = \left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \tag{2}$$

and

$$\tau y_t = g_{1t} + g_{2t} \tag{3}$$

[2] As in Devarajan et al (1996), labor does not enter directly in the production function.

where $1 \geq \alpha_1 \geq 0, \zeta_1 \in (-1,0) \cup (0,+\infty),\ 0 < \tau < 1$.

The capital stock, k, follows a standard law of motion:

$$\dot{k} = (1-\tau)y - c \tag{4}$$

The representative agent chooses consumption, c, and capital, k, to maximize the expected discounted value of his utility. The utility function has the isoelastic form of a CRRA (constant relative risk aversion) function:

$$\int_0^{+\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{5}$$

with $\sigma > 0,\ \sigma \neq 1,\ \rho > 0$.

Taking all this into account, the representative agent's optimization problem might be written as:

$$(P)\begin{cases} Max & \int_0^{+\infty} \frac{c_t^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \\ s.t. & \dot{k}_t = (1-\tau)y_t - c_t \\ & y_t = A_t \left[\alpha k_t^{-\zeta} + (1-\alpha) x_t^{-\zeta}\right]^{-\frac{1}{\zeta}} \\ & x_t = \left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \\ & \tau y_t = g_{1t} + g_{2t} \\ & 1 \geq \alpha > 0,\ 1 \geq \alpha_1 \geq 0,\ \zeta, \zeta_1 \in (-1,0) \cup (0,+\infty),\ 0 < \tau < 1, \\ & 0 < \sigma,\ \sigma \neq 1,\ \rho > 0 \end{cases}$$

where the functions A, c, k, y, g_1 and g_2 are defined on $[0,+\infty)$ with positive values (i.e : $[0,+\infty) \rightarrow (0,+\infty)$).

Notice that

$$y_t = A_t \left[\alpha k_t^{-\zeta} + (1-\alpha)\left[\alpha_1 g_{1t}^{-\zeta_1} + (1-\alpha_1) g_{2t}^{-\zeta_1}\right]^{\frac{\zeta}{\zeta_1}}\right]^{-\frac{1}{\zeta}} \tag{6}$$

which is also a CES function.

2.1 Theoretical Results

The solution of problem (P) under the previous parametrization, and in the special case of a Cobb-Douglas production function, allows us to derive our major findings. They are reported in the following sequence of Lemmas and Theorems.

Lemma 1. *There exists* $\phi : [0, +\infty) \to [0, 1]$ *such that* $g_{1t} = \phi_t \tau y_t$ *and* $g_{2t} = (1 - \phi_t)\tau y_t$ $\forall t \in [0, +\infty)$.

Proof. Follows from the fact that $\tau y_t = g_{1t} + g_{2t}$ $\forall t \in [0, +\infty)$. □

Theorem 1. *If* $Im(\phi) \subset (0, 1)$, *then there exists* $\delta : [0, +\infty) \to (0, 1) \cup (1, +\infty)$ *defined by*

$$\delta_t = \left[\alpha_1 \phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1}\right]^{\frac{\zeta}{\zeta_1}} \tag{7}$$

such that:

1. $x_t = \tau \delta_t^{-\frac{1}{\zeta}} y_t$.
2. $\delta_t \in (0, 1)$ *when* $\zeta \in (-1, 0)$.
3. $\delta_t > 1$ *when* $\zeta > 0$.

Proof.

1. By definition of the function x.

$$\begin{aligned}
x_t &= \left[\alpha_1 g_{1t}^{-\zeta_1} + (1 - \alpha_1) g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \\
x_t^{-\zeta_1} &= \alpha_1 g_{1t}^{-\zeta_1} + (1 - \alpha_1) g_{2t}^{-\zeta_1} \\
x_t^{-\zeta_1} &= \alpha_1 \phi_t^{-\zeta_1} \tau^{-\zeta_1} y_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} \tau^{-\zeta_1} y_t^{-\zeta_1} \\
x_t &= \left[\alpha_1 \phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1}\right]^{-\frac{1}{\zeta_1}} \tau y_t \\
x_t &= \tau \delta_t^{-\frac{1}{\zeta}} y_t
\end{aligned}$$

   The aggregate public spending, x_t, is generated by $\tau \delta_t^{-\frac{1}{\zeta}}$ multiplied by the output of the economy. A special case emerges when $\zeta_1 \to 0$, which implies that $\delta = 1$, $x_t = \tau y_t$, and $x_t = g_{1t} + g_{2t}$.

   The third equation follows from the previous Lema.

2. If $\zeta_1 \in (-1, 0)$, then $0 < \phi_t^{-\zeta_1} < 1$ and $0 < (1 - \phi_t)^{-\zeta_1} < 1$. So, $0 < \alpha_1 \phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} < 1$. The statement follows because $\frac{\zeta}{\zeta_1} > 0$. If not $\zeta_1 > 0$. Here, $\phi_t^{-\zeta_1} > 1$ and $(1 - \phi_t)^{-\zeta_1} > 1$. So, $\alpha_1 \phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} > 1$. The statement follows because $\frac{\zeta}{\zeta_1} < 0$.

3. If $\zeta_1 \in (-1, 0)$, then $0 < \phi_t^{-\zeta_1} < 1$ and $0 < (1-\phi_t)^{-\zeta_1} < 1$. So, $0 < \alpha_1 \phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} < 1$. The statement follows because $\frac{\zeta}{\zeta_1} < 0$. If not $\zeta_1 > 0$. Here, $\phi_t^{-\zeta_1} > 1$ and $(1-\phi_t)^{-\zeta_1} > 1$. So, $\alpha_1 \phi_t^{-\zeta_1} + (1-\alpha_1)(1-\phi_t)^{-\zeta_1} > 1$. The statement follows because $\frac{\zeta}{\zeta_1} > 0$.

□

Theorem 2. *If $Im(\phi) \subset (0,1)$, then there exists $\theta : [0, +\infty) \to (0, +\infty)$ defined by*

$$\theta_t = \left[\frac{(A_t \tau)^\zeta - (1-\alpha)\delta_t}{\tau^\zeta \alpha} \right]^{-\frac{1}{\zeta}} \tag{8}$$

such that $k_t = \theta_t y_t \ \forall t \in [0, +\infty)$. If $A_t = 1$, then $\theta_t > 1$.

Proof. By definition of function y.

$$\begin{aligned}
y_t &= A_t \left[\alpha k_t^{-\zeta} + (1-\alpha) x_t^{-\zeta} \right]^{-\frac{1}{\zeta}} \\
y_t^{-\zeta} &= \alpha (A_t k_t)^{-\zeta} + (1-\alpha)(A_t x_t)^{-\zeta} \\
y_t^{-\zeta} &= \alpha (A_t k_t)^{-\zeta} + (1-\alpha)(A_t \tau)^{-\zeta} \delta_t y_t^{-\zeta} \\
\alpha A_t^{-\zeta} k_t^{-\zeta} &= \left[1 - \tfrac{1-\alpha}{(A_t \tau)^\zeta} \delta_t \right] y_t^{-\zeta} \\
k_t &= \left[\tfrac{(A_t \tau)^\zeta - (1-\alpha)\delta_t}{\alpha \tau^\zeta} \right]^{-\frac{1}{\zeta}} y_t \\
k_t &= \theta_t y_t
\end{aligned}$$

Thus, each unit of private capital generates $\frac{1}{\theta}$ units of output in the economy. In other words, $\frac{1}{\theta}$ represents the private-capital productivity.

The third equation follows from the substitution of x_t. Note that $(A_t \tau)^\zeta - (1-\alpha)\delta_t > 0$, because functions k and y are positive.

Now, we consider the particular case in which $A_t = 1$.

If $\zeta \in (-1, 0)$, then

$$\begin{array}{lllllll}
0 & < & \delta_t & < & 1 & < & \tau^\zeta \ \ \forall t \geq 0 \\
0 & < & (1-\alpha)\delta_t & < & 1-\alpha & < & (1-\alpha)\tau^\zeta \ \ \forall t \geq 0
\end{array}$$

This implies that

$$\begin{array}{lllll}
0 & < & \alpha \tau^\zeta & < & \tau^\zeta - (1-\alpha)\delta_t \ \ \forall t \geq 0 \\
0 & < & 1 & < & \left[\frac{\tau^\zeta - (1-\alpha)\delta_t}{\alpha \tau^\zeta} \right] \ \ \forall t \geq 0 \\
0 & < & 1 & < & \theta_t \ \ \forall t \geq 0
\end{array}$$

If not ($\zeta > 0$), then

$$\begin{array}{ccccccc} 0 & < & \tau^{\zeta} & < & 1 & < & \delta_t \ \ \forall t \geq 0 \\ 0 & < & (1-\alpha)\tau^{\zeta} & < & 1-\alpha & < & (1-\alpha)\delta_t \ \ \forall t \geq 0 \end{array}$$

This implies that

$$\begin{array}{ccccc} 0 & < & \tau^{\zeta} - (1-\alpha)\delta_t & < & \alpha\tau^{\zeta} \ \ \forall t \geq 0 \\ 0 & < & \left[\frac{\tau^{\zeta}-(1-\alpha)\delta_t}{\alpha\tau^{\zeta}}\right] & < & 1 \ \ \forall t \geq 0 \\ 1 & < & \theta_t \ \ \forall t \geq 0 & & \end{array}$$

Note that all quantities are positive. The last inequality follows because $-\frac{1}{\zeta} < 0$.

□

This means that this model collapses to a traditional Ak growth model. In the special case of a Cobb-Douglas production function, where $\zeta = 0$ and $\zeta_1 = 0$, we have that both $\delta_t = 1$ and $\theta_t = 1$. From now on, we no longer assume that $A_t = 1$.

Theorem 3. *If $Im(\phi) \subset (0,1)$ and $c : [0, +\infty) \to (0, +\infty)$ is a continuously differentiable function, then the equation for the growth rate consumption is:*

$$\lambda_t = \frac{\dot{c}_t}{c_t} = \frac{1-\tau}{\sigma\theta_t} - \frac{\rho}{\sigma} \tag{9}$$

Proof. Following Lemma 2, we have that the dynamical equation for k might be rewritten as $\dot{k}_t = \left[\frac{1-\tau}{\theta_t}\right] k_t - c_t$. Thus, problem (P) is nothing else than a variational calculus problem, where:

$$L(t, k_t, \dot{k}_t) = \left[\frac{\left[\frac{1-\tau}{\theta_t}k_t - \dot{k}_t\right]^{1-\sigma} - 1}{1-\sigma}\right] e^{-\rho t}. \tag{10}$$

Here, $\frac{\partial L}{\partial k} = \left[\frac{1-\tau}{\theta_t}\right] c_t^{-\sigma} e^{-\rho t}$ and $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{k}}\right] = [\sigma c_t^{-1-\sigma}\dot{c}_t + \rho c_t^{-\sigma}]e^{-\rho t}$. The statement follows from the Euler equation. □

Corollary 1. *If $\frac{1-\tau}{\rho} \leq \theta_t$, then $\lambda_t \leq 0$.*

Corollary 2. *If $\frac{1-\tau}{\rho} > \theta_t$, then $\lambda_t > 0$.*

Theorem 3 states that the growth rate of consumption in this economy might be positive or negative. Its sign, according to Corollaries 1 and 2, depends on $\frac{1-\tau}{\rho}$ and θ_t. In order to keep a positive growth rate of consumption, $\lambda_t > 0$, the economy should be characterized by low degrees of impatience and tax rate, such that $\frac{1-\tau}{\rho} > \theta_t$. In this environment, the representative agent would be willing to transfer consumption across time and the after-tax income would be sufficient to allow for this transference. In the special case of a Cobb-Douglas, we have that $\lambda_t > 0$ only if $\tau + \rho < 1$, given that $\lambda_t = \frac{(1-\tau-\rho)}{\sigma}$ in this case. Next, in Lemma 2, we define the optimal share for productive (and unproductive) government expenditure also as a function of the economy structural parameters.

Lemma 2. *If $Im(\phi) \subset (0,1)$, then θ as a function of ϕ attains its minimum value at* $\phi^* = \frac{\alpha_1^{\frac{1}{\zeta_1+1}}}{(1-\alpha_1)^{\frac{1}{\zeta_1+1}}+\alpha_1^{\frac{1}{\zeta_1+1}}}$.

Proof. Since,

$$\theta_t = \left[\frac{(A_t\tau)^\zeta - (1-\alpha)\delta_t}{\alpha\tau^\zeta}\right]^{-\frac{1}{\zeta}} \tag{11}$$

then

$$\frac{\partial\theta}{\partial\phi} = -\frac{1-\alpha}{\alpha\tau^\zeta}\theta_t^{\zeta+1}\delta_t^{1-\frac{\zeta_1}{\zeta}}\left[\alpha_1\phi_t^{-\zeta_1-1} - (1-\alpha_1)(1-\phi_t)^{-\zeta_1-1}\right] \tag{12}$$

So, $\frac{\partial\theta}{\partial\phi} = 0$ at $\phi^* = \frac{\alpha_1^{\frac{1}{\zeta_1+1}}}{(1-\alpha_1)^{\frac{1}{\zeta_1+1}}+\alpha_1^{\frac{1}{\zeta_1+1}}}$.

We point that if $\phi \in (0, \phi^*)$, then $\frac{\partial\theta}{\partial\phi} < 0$. Analogously, if $\phi \in (\phi^*, 1)$, then $\frac{\partial\theta}{\partial\phi} > 0$. Thus, the statement follows. □

Essentially, the optimal share of the productive public expenditure, ϕ^*, depends on α_1 and ζ_1. In the special case of a Cobb-Douglas production function, this optimal share simplifies to $\phi^* = \alpha_1$. This is intuitive because the higher the elasticity of the productive expenditure in the aggregate public spending, x_t, the higher will be the optimal share of the productive expenditure.

Theorem 4. *The maximizer of the growth rate consumption is the minimizer of θ as a function of ϕ.*

Proof. Since the growth rate consumption is

$$\lambda_t = \frac{\dot{c}}{c} = \frac{1-\tau}{\sigma\theta_t} - \frac{\rho}{\sigma} \tag{13}$$

The statement follows because the maximum value of λ is achieved at the same ϕ^* that θ achieves its minimum value with respect to ϕ. $\square$

Next, we try to find the optimal τ which is compatible with the maximum growth rate of consumption. Notice that, by problem (P), this optimal τ also corresponds to the optimal size of the government expenditure in the economy.

Theorem 5. *Function k increases when τ decreases.*

Proof. Since $k_t = \theta_t y_t$, the statement follows because

$$\frac{\partial\theta}{\partial\tau} = -\frac{1-\alpha}{\alpha}\left[\frac{\theta_t}{\tau}\right]^{\zeta+1}\delta_t \tag{14}$$

$\square$

Theorem 6. *The maximizer of the growth rate consumption as a function of τ is $\tau^* = \left[\frac{(1-\alpha)\delta_t}{A_t^{\zeta}}\right]^{\frac{1}{\zeta+1}}$.*

Proof. Note that

$$\frac{\partial\lambda}{\partial\tau} = \frac{-\sigma\theta_t - (1-\tau)\sigma\frac{\partial\theta}{\partial\tau}}{\sigma^2\theta_t^2} \tag{15}$$

But, from Theorem 5 we have that $\frac{\partial\theta}{\partial\tau} = -\frac{1-\alpha}{\alpha}\left[\frac{\theta_t}{\tau}\right]^{\zeta+1}\delta_t$.

So,

$$\frac{\partial\lambda}{\partial\tau} = \frac{-A_t^{\zeta}\tau^{\zeta+1} + \delta_t(1-\alpha)}{\sigma\tau\theta_t((A_t\tau)^{\zeta} - (1-\alpha)\delta_t)} \tag{16}$$

The statement follows, because for each $\tau < \tau^*$ the function λ increases. Analogously, for each $\tau > \tau^*$ the function λ decreases. In the special case where $\zeta \in (-1, 0)$, a positive variation in A_t leads to an increase in $\tau*$. This suggests that a higher technological progress allows for a higher level of optimal taxation, provided that all other variables are kept unchanged. $\square$

Theorem 7. *The growth rate consumption function λ is an increase function with respect to A.*

Proof. This follows from the fact that

$$\frac{\partial \lambda}{\partial A} = \left[\frac{1-\tau}{\alpha\sigma}\right](\theta_t A_t)^{\zeta-1} \tag{17}$$

□

3 Empirical Evidence

3.1 Econometric Model

We estimate, by nonlinear least squares, the aggregate CES function which emerges from the combination of the previous equations (1) and (2). In a panel data environment, considering the data set that will be used in the estimation, it might be written as:

$$y_{jt} = A_{jt}\left[\alpha k_{jt}^{-\zeta} + (1-\alpha)\left[(1-\eta_1)g_{1jt}^{-\zeta_1} + \eta_1 g_{2jt}^{-\zeta_1}\right]^{\frac{\zeta}{\zeta_1}}\right]^{-\frac{1}{\zeta}} + \varepsilon_{jt} \tag{18}$$

This equation is equivalent to the one that emerges from the theoretical model with $(1-\eta_1) = \alpha_1$ being the share public spending in investment. The structural parameter α defines the share of private capital in the output while $(1-\alpha)$ represents the share of the aggregate government spending in the output. In addition, η_1 is the share of current spending and $(1-\eta_1)$ is the share of public investment in the aggregate government spending[3]. The parameters ζ and ζ_1 are elasticities[4] and ε is the additive random error.

The heterogeneous technological progress in each state is A_{jt}. Following Duffy and Papageorgiou (2000), it is described by:

$$A_{jt} = exp(\gamma p_i + \nu t) \tag{19}$$

We use the cumulative distribution of patents across the Brazilian states to create a dummy variable for the $i\%$ group of the least productive ones in terms of number of registered patents. These states are considered as having low degree of technological progress. They will have an output smaller than the average whenever $\gamma < 0$.[5] Given that the technological progress might change over time, ν accounts for the estimation of this temporal effect.

[3] $(1-\eta_1)$ is the share of public investment because g_1 does not include expenses with interest rate and debt rollover.

[4] Because it is a CES, the constant elasticities are given by $\psi = \frac{1}{1+\zeta}$ and $\psi_1 = \frac{1}{1+\zeta_1}$.

[5] We considered $i = 1\%$ and $i = 5\%$ in the estimation, but the results were similar. We also used other variables, such as number of patents, average of registered patents and average of registered patents in the last 5 years. However, the estimations did not converge. Thus, we chose to work only with $i = 5\%$.

Finally, we considered variables in both levels and per capita terms. Thus, we estimate four regression models, as described below:

1. Model (A): With no difference in technological progress, such that $A_{jt} = 1$ for all states;
2. Model (B): With difference in technological progress, such that $i = 5\%$ of the states that least registered patents;
3. Model (C): Similar to the model (A), but with per capita variables;
4. Model (D): Similar to the model (B), but with per capita variables.

3.2 Data and Variables

We use a balanced panel composed by the 27 Brazilian states in the period from 2004 to 2010 with annual data, totalizing 189 observations.[6] The nominal variables were deflated by the wide consumer price index (Indice Nacional de Preços ao Consumidor Amplo - IPCA), which is calculated by the Brazilian Institute of Geography and Statistics (IBGE) and used by the Central Bank of Brazil in the inflation targeting regime. Each variable is describe in sequence.

y: Gross domestic product (GDP) of the Brazilian states released by Ipeadata[7].

g_1: Government spending in investment by each Brazilian state, which consists of total capital spending minus payment of interest rate and debit amortization. The source also is Ipeadata.

g_2: Government spending on costing (or current spending) by each Brazilian state, also collected from the Ipeadata.

k: Stock of private capital of each Brazilian state, computed according to the procedure proposed by Sanches and Rocha (2010).

The technological progress of each Brazilian state was measured by the amount of registered patents provided by the National Institute of Intellectual Property (Instituto Nacional da Propriedade Industrial - INPI). We create a dummy variable, p_5, to represent the 5% of the Brazilian states that least registered patents in each time period. Thus, $p_5 = 1$ when state j is part of these 5% that least registered patents according to the cumulative distribution of patents and $p_5 = 0$ otherwise.

[6] The sample is restricted to this period because of the data availability for the computation of the private capital k.

[7] www.ipeadata.gov.br.

3.3 Empirical Results

Initially, we tested the panel data for the presence of unit root. We applied tests due to Levin-Lin-Chu (LLC), Im-Pesaran-Shin (IPS), Fisher-ADF, Hadri, and Pedroni (1999). The results indicated that the panel is stationary at the 5% significance level. The non-linear estimation of equation (18) was carried out in Stata version 11. Initial values of the parameters were set at 0.0001 in order to allow for convergence to either positive or negative values.

Due to individual heterogeneity of each Brazilian state, which might lead to heteroscedasticity in the residuals, we performed a correction for robust standard errors in clusters. However, in all results reported in Table 1, the parameters that were statically significant at the 1% level maintained this significance even without that correction.

The parameter ν in equation (19), which identifies the time effect on technological progress, was not statistically significant in any model. This might be due to short time horizon of the panel data. Thus, the time effect was taken out from the estimations and only estimated values for γ are reported in Table 1. As mentioned before, the first two estimated models refer to variables in levels while the last two use per capita variables. In addition, models A and C assume no technological progress across the Brazilian states ($A = 1$), while models B and D estimates technological progress according to equation (19) with $\nu = 0$.

The share of private capital in the total output, α, was estimated above 65% in all models, indicating that private capital is more important than the government's compound spending for the output of the Brazilian states. This high estimated value might also capture the effect of labor in the output, which is not explicitly modeled in the production function (1) that is based in Devarajan et al (1996). The negative estimated value for ζ points out that private investment and government spending are complimentary inputs in the production. This might suggest that there is a crowding in effect between government spending and private investment across the Brazilian states.[8]

Considering the composition of government spending, η_1 is always above 85% meaning that the government spends a larger fraction of the total public expenditure on current spending (g_2) than in public investment (g_1). This finding is in line with the fact that public investment is still a small fraction of public spending in the Brazilian economy. The coefficient ζ_1 was not statistically different from zero at the 5% significance level, suggesting that the

[8] The elasticity of substitution, ψ, ranges from 1.403 in model A to 1.468 in model D.

Table 1: Non-linear estimation

Parameters	(A)	(B)	(C)	(D)
α	0.660***	0.659***	0.694***	0.689***
	(0.074)	(0.072)	(0.075)	(0.049)
ζ	-0.287***	-0.290***	-0.303***	-0.319***
	(0.066)	(0.065)	(0.068)	(0.049)
η_1	0.887***	0.853***	0.989***	0.861***
	(0.132)	(0.163)	(0.042)	(0.077)
ζ_1	0.356	0.210	1.270	0.203
	(0.730)	(0.708)	(1.528)	(0.340)
γ	NA	-0.634***	NA	-0.680***
		(0.086)		(0.101)
R-squared	0.965	0.972	0.968	0.990
Adj. R-squared	0.964	0.971	0.967	0.990

Source: Estimated by the authors. Notes: ***, **, and * indicate that the estimated coefficient is statistically significant at the 1, 5, and 10% levels, respectively.

public spending composition, x_t, might be represented by a Cobb-Douglas function, which is a special case of the general CES assumed in equation (2).

The Brazilian states classified as having lower technological progress according to the amount of registered patents presented a negative and statistically significant value for γ. This means that they have a smaller output than the other states, which are in the group that registered more than 5% of the patents in each time period. Thus, the states contained in p_5 are expected to have an output which is, on average, about 51 to 53% smaller than the states that are outside p_5, according to models (B) and (D) respectively.[9] This finding is common in the literature, where the less technologically developed economies are also the ones with lower levels of production.

To find the growth rate of consumption λ and the other compound parameters implied by the theoretical model, we need to set values for the risk aversion coefficient σ and the intertemporal discount factor, ρ, in addition to the estimated coefficients from Table 1. Notice that the preference of the government for investment spending is $\alpha_1 = (1-\eta_1)$. The values for $\sigma = 4.89$ and $\rho = 0.123$ were obtained from from Issler and Piqueira (2000).[10] With

[9] These values were computed by making $A_{jt} = exp(\gamma p_5)$, where γ = -0.634 and -0.680 for the states that belong to p_5 in models B and D, respectively, according to Table 1.

[10] Issler and Piqueira (2000) reported the intertemporal discount rate as $\beta = 0.89$. To

these values, we can find the average growth rate of consumption, $\overline{\lambda}$, the ratio of private capital to output, $\overline{\theta}$, and the deviations from the predicted $\overline{\lambda}$ to the observed growth rate from the data, as reported in Table 2.

Table 2: Compound structural parameters

Parameters	(A)	(B)	(C)	(D)
α_1	0.113	0.147	0.011	0.139
$\overline{\theta}$	2.151	3.453	1.920	3.148
$\overline{\lambda}$	0.060	0.033	0.070	0.040
$Mean\ deviation$	-0.010	0.017	-0.033	0.003
$Mean - square\ deviation$	0.000	0.001	0.001	0.001

Source: Calculated by the authors.

The inverse of $\overline{\theta}$ measures the productivity of the private capital in the production. Thus, the lower $\overline{\theta}$ the more productive is the private capital. Table 2 indicates that this productivity ranges from 0.52 to 0.29, depending on the estimated model.

Comparing the estimated models with and without technological progress, model (A) yields the smallest mean-squared deviation from $\overline{\lambda}$ with respect to the observed consumption growth rate from the data. Except for model (C), the values of the estimated parameters are similar across the alternative models. This meas that the estimated structural parameters are quite stable across the models.

The optimal fraction of investment spending, $\phi*$, and optimal tax rate, $\tau*$, that maximize consumption growth, λ, are reported in Table 3. The short time horizon of the sample might have biased the impact of the government spending in investment over the economic growth of the Brazilian states, as measured by $\phi*$.[11] Given that the average total tax revenue is around 16% of the GDP for the Brazilian states, according to the data for the 2004 to 2010 period available at ipeadata.gov.br, one might argue that $\overline{\tau} < \tau*$ and some states have average taxation below the optimum level.

Because ζ_1 is not statistically different from zero, we have that $\phi* = \alpha_1$. In this case, only with the optimal levels of investment spending, $\phi*$, and tax rate, $\tau*$, it is not possible to access how the consumption growth rate

find the intertemporal discount factor, ρ, we use $\beta = \frac{1}{1+\rho}$, which yielded $\rho = 0.123$. Other authors, such as Catalão and Yoshino (2006), Costa and Carrasco (2015), and Faria and Ornelas (2015) have found similar values for σ and ρ.

[11] Divino and Silva Jr. (2012) found that the optimal share of public spending in capital is 32% for high-income, 23% for middle-income, and 19% for low-income Brazilian municipal districts.

Table 3: Optimum levels of ϕ and τ.

Parameters	(A)	(B)	(C)	(D)
$\phi*$	0.113	0.147	0.011	0.139
$\tau*$	0.220	0.195	0.183	0.155

Source: Computed by the authors.

depends on these parameters. To do so, we need to compute the partial derivatives reported in Tables 4 and 5.

Table 4: Sensitivity of θ with respect to ϕ and τ.

Parameters	(A)	(B)	(C)	(D)
$\partial\theta/\partial\phi$	0.168	0.086	0.061	0.102
$\partial\theta/\partial\tau$	-4.889	-6.338	-3.534	-4.613

Source: Computed by the authors.

The ratio of public spending in investment, ϕ, according to Lemma 2 is above the optimal level because $\partial\theta/\partial\phi > 0$. This result implies that an increase in ϕ leads to a decrease in the productivity of the private capital and so to a decrease in in the growth rate of consumption, λ. According to Theorem 4, the maximization of λ as a function of ϕ occurs when θ is minimum. This might also mean that public spending in costing, g_2, is below the optimal level.

An increase in the tax rate, τ, decreases the private capital relatively to the output of the economy. This is a classic result given that an increase in taxation will reduce the amount of private capital available in the economy. However, the effect of a higher τ on λ is not obvious because the productivity of the private capital increases.

Table 5: Sensitivity of λ with respect to τ and A.

Parameters	(A)	(B)	(C)	(D)
$\partial\lambda/\partial\tau$	0.087	0.064	0.067	0.046
$\partial\lambda/\partial A$	0.104	0.094	0.112	0.103

Source: Computed by the authors.

In fact, as shown in Table 5, $\partial\lambda/\partial\tau > 0$ for the Brazilian states. The effective taxation is below the optimal level, suggesting that there is margin for a tax increase without harming economic growth of the economy.

Another result from Table 5 is the positive effect of changes in technological progress to the consumption growth. This is expected once an increase in technology has a direct effect on output, which leads to a rise in consumption. This effect is very similar across the alternative models, as illustrated in Table 5.

4 Concluding Remarks

The objective of this paper was to investigate both the optimal size of the government in the economy and the optimal shares of productive and unproductive public expenditures in the aggregate government spending. We expanded the model by Devarajan et al (1996) to include an exogenous technological progress in a more general constant elasticity of substitution (CES) production function. We showed how those optimal shares depend on the structural parameters of the economy and provided empirical evidence by using a balanced panel data for the Brazilian states in the recent period.

The Cobb-Douglas and Ak production functions are obtained as special cases of the general CES specification. In the special case of Cobb-Douglas production function, economic growth requires that the sum of tax rate and intertemporal discount factor in the utility be strictly smaller than one. In addition, the optimal public spending that maximizes consumption growth is given by the share of public spending allocated on capital.

In the general case, however, the optimal taxation depends on the whole set of structural parameters. In particular, the technological progress has a direct effect on the optimal level of taxation while the share of private capital in total production has a negative effect on that taxation. This theoretical finding coincided with the empirical results estimated for the Brazilian economy.

The estimated parameters for the Brazilian states suggest that, on average, the share of private capital in the total production is 0.66 while the share of total government spending is 0.34. The estimated elasticity of substitution between these two inputs ranged from 1.40 to 1.45, depending on the model specification. Thus, private capital has a much higher share than government spending in the production.

Devarajan et al. (1996) argue that developing countries, due to the low economic dynamism, require a significant fraction of government spending allocated to costing. This finding was observed for the Brazilian economy, were about 85% of the total public spending was in costing and only 15% was in public investment. This result is in line with the widely spread consensus that public investment is still a small fraction of public spending in the Brazilian economy.

Taking as a whole, the government spending is below the optimal level. Thus, it is possible to increase the growth rate of consumption by rising the government spending under a balanced public budget. The estimated optimal taxation by the model that best fitted the data was 22%, while the average taxation observed from the data for the Brazilian states is around 16%. Thus, there is space for increasing taxation, and so for rising government spending, without hurting the economic growth.

For future research, it would be interesting to include public debt in the government budget constraint along with its dynamics in an intertemporal environment. It would be also interesting to expand the empirical analysis to a panel of countries with data for the pre and post international financial crisis. Some of these suggestions are object of our current research.

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